

# Reem-B: an autonomous lightweight human-size humanoid robot

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**Abstract**— This paper introduces the humanoid robot Reem-B, built by Pal Technology as a research platform in the field of service robots. The idea is to produce robots that can help humans and cohabit their environments. For this purpose, the body plan, sensory and actuator system of the robot, as well as its cognitive abilities must be designed to perform real-world tasks including dynamic walking, interaction with people or object recognition and manipulation. Reem-B achieves this scope by using two legs, two strong arms with fingered hands, and a software suite that controls all its degrees of freedom, coordinating them with vision and auditory systems. The main difference with other humanoids of its size is its level of autonomy. Autonomy in this robot has been improved from other robots at three different levels: with an increased battery life (estimated twice of the competitors), with the ability to autonomously navigate in indoor environments while avoiding obstacles, and by integrating all the control software within the robot itself.

## I. INTRODUCTION

Humanoid robotics is a field of research that has incredibly grown in the last decade [1]. One of the motivations for this growth is the increased interest on developing a service robot which can assist humans in their daily activities [2]. Applications of such type of robot range from homework assistance, to perform dangerous jobs, or to take elderly care.

To develop this kind of autonomous robots, it is necessary to master many different subjects in the fields of mechanics, electronics and automation control. Also, from the software point of view, it is required the integration of computer vision, autonomous navigation, voice interaction and learning systems [3]. All those abilities must be integrated within a single robotic system, being this integration a very hard process due to the different nature of each part involved [4], [5], [6], [7].

Even if the goal of having complete autonomous humanoid robots is still far from achieved, several steps have been made towards it. For instance, several humanoid robots have been created in the last decade for their interaction with people, being one of the most famous the Asimo [8]. In the same line we can find the HRP-3 [9], Hubo [10], P-chan [11], Lola [12] and Wabian [13] robots. However, even if all those robots have a more or less adequate size and equipment for its use in home environments, they all lack the ability of being autonomous, understanding by autonomous a robot that can move by itself around a space and interact upon it, without requiring human intervention to accomplish its assigned tasks [14]. For those robots, their autonomy level is really reduced,

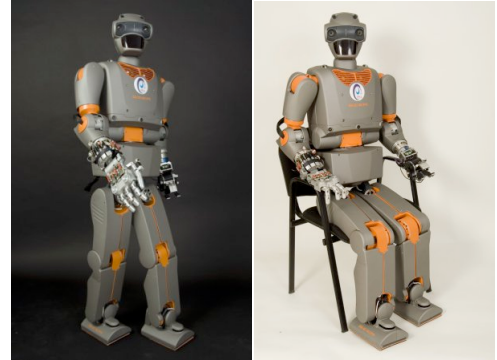


Fig. 1. Images of the Reem-B robot walking (left) and sat down (right)

because are remote controlled, have a short battery duration or some the control algorithms are executed on an external computer to which the robot is linked.

An example of a real autonomous humanoid robot is QRIO by Sony [15]. This robot is one of the few robots that integrate some level of autonomy with humanoid characteristics, including a vision based autonomous navigation system [16]. Following a similar line, the Nao Robot by Aldebaran presents autonomous abilities which have made it the selected platform for the Robocup soccer competition. However, despite the autonomous abilities of those two robots, their small size (no more that 60 cm) does not allow their use in real world tasks which require interaction with most of the stuff found in human environments, like stairs, chairs, tables, and all kind of small human stuff. The same problem applies to other humanoid robots like the iCub [17] or the Hoap series [18].

As an opposite reaction to those robots, Reem-B has been physically designed to include all the abilities required to live on a human environment and act upon it in a similar way as humans do. On top of that, Reem-B is, to our knowledge, one of the first robots in the world that integrates some degree of autonomy within a human sizable robot.

This paper describes the inner workings of Reem-B in the following sections: section 2 contains a list of requirements for the Reem-B robot. Section 3 describes the different hardware and software components that meet previous section requirements, and section 4 includes some tests performed by the robot, showing its abilities.

## II. DESIGN CONCEPT AND SPECIFICATIONS

The requirements for a home robot are those of a robot that must share its environment with humans, and perform

its job interacting with humans' stuff. This means that it must have a reasonable size, high enough to reach tables and drawers. It must be able to move around a home environment while avoiding obstacles, and carry small objects. It must also include some cognitive abilities that allow it to interact with humans, including face and voice recognition, as well as a minimal capacity to recognize objects and manipulate them.

Even if the available body designs for such kind of robot can be diverse, one of the most suitable for the interaction in houses or offices is that with a humanoid shape, that is, a torso with arms that may optionally include legs. A legged robot, though, may increase its operation range in human environments allowing it to climb stairs, overcome ground level differences or greater changes in the type of terrain. Hence, in order to construct a more suitable robot, the Reem project is based on a legged humanoid.

About the size of the robot, three different conditions are required: first, the robot should be able to reach typical human stuff. Second, the control of the robot movement should be as simple as possible (difficulty increases with the robot height). And third, the battery of the robot should last as long as possible. Battery life decreases with the increase of the robot size. We took a compromise solution between the three and designed Reem-B with a height of 1.47 meters and about 60 Kg of weight including batteries.

A home robot must be able to walk on flat surfaces at a human speed. Speeds between 0.7-1.0 Km/h seem reasonable to move around a house. Reem-B finally achieved peak speeds of 1.5 Km/h. Additional mobility abilities like climbing stairs or sitting down on chairs may be interesting for a robot of its size, and have been included in Reem-B.

The robot for home must be strong enough to manipulate and transport light objects, like books, bottles or phones. For this purpose, Reem-B has been designed to have a 6 Kg arm payload. As an optional feature, a home robot should be able to manipulate small stuff, by grasping it and carrying it to other place. For this reason, Reem-B has been equipped with a four fingered hand, which has been successfully used in different tasks like playing chess, or grasping cans and bottles.

Related to its cognitive abilities, the robot must be able to recognize faces, and objects, given a provided database of objects. It has to be able to associate faces with data, like names, and agenda memos. Humans must be able to interact with the robot in a natural way, using voice.

Finally, the robot must be autonomous. And by autonomy we consider three main aspects: first, the battery must provide a long working time. Second, the robot must be able to localize itself within a given environment, and move autonomously on it. And third, all the computation and control must be performed onboard the robot. Reem-B integrates all those aspects what makes of it one of the most autonomous robots of its size.

### III. HARDWARE AND SOFTWARE COMPONENTS

The Reem team has focused on two main points: first, the research and development of the mechanical structure of the robot. This focus on the mechanics allowed to create a humanoid robot with a lower weight and higher autonomy than existing ones, while keeping the necessary stiffness. Second, the integration of different software technologies into a single system, achieving cognitive and autonomous movement abilities.

#### A. Mechanical design

The mechanical design of the robot has paid special attention to find a compromise between size, stiffness, battery consumption and weight.

1) *Physical structure:* The physical structure includes the legs and the torso of the robot. Those are the parts that will contain the rest of the robot elements.

In order to produce a stable behavior and accurate dynamic movement, the body is required to have a stiff mechanical structure, which reduces the oscillations amplitude. Usually, stiffness comes at the cost of weight, which ends in higher levels of power consumption. To avoid this situation as much as possible, a tradeoff between stiffness and weight has been achieved by using FEM analysis [19]. The result is an aluminum frame which weights no more than 35 kg with stiffness enough to walk stable while carrying a 6 kg load per arm.

The kinematic structure of Reem-B and the number of degrees of freedom (DoF) is very similar to other humanoid robots and is summarized in Table I; it must be noted that hip joints have 3 DoF intersecting on a single point. This structure is more similar to the human one, where the femur bone is connected to the ilium with a spherical joint. Additionally, the waist has been equipped with two DoF, which increase mobility of the upper body.

Most of the reducers used on the robot's joints are Harmonic Drives (HD). The HD technology has several advantages compared to planetary gear-heads: despite their low efficiency at higher velocities, they have nearly zero backlash, and one of the best torque to weight ratio. In addition, HD are back-drivable, an important feature especially on the legs, which experience impacts with the ground during the normal walking gate.

The motors used are all brush-less. This kind of motors have been selected instead of the typical DC brushed motors because of their higher power/weight ratio, efficiency, longer life and less electro-magnetic emission. The drawbacks are the necessity of some extra cabling (because of the hall sensor integrated in the motors) and a more complicated control and driving electronic.

2) *Arm design:* Arms are based on modular elements created by the Reem team for the purpose. Each module contains on a single piece, motor bearings, reduction and control board, and does not require additional elements.

Each arm is composed by five modules, providing therefore the arm with five DoF. Additionally, each arm is



Fig. 2. Reem-B climbing stairs sequence



Fig. 3. Reem-B walking sequence

Body part	Degrees Of Freedom
Legs	6x2
Arms	5x2
Waist	2
Head	3
Hands	1x2+1x11

TABLE I

TABLE OF DEGREES OF FREEDOM. THE ROBOT HAS A TOTAL OF 40 DOF.

equipped with a 6-axis force sensor for the detection of external forces applied to them.

3) *Hand design:* Reem-B is equipped with one gripper hand and one four fingered hand. The gripper hand is based on a gripper with two DoF and IR sensors that detect whether there is something between the gripper fingers. This hand is used to take heavy stuff like filled water bottles, books or TV remotes.

The fingered hand has four fingers with a total of 11 DoF. A complete system based on tendons allows the control of the hand to grasp small things. The hand is also equipped with IR and pressure sensors on each finger.

#### B. Electronic hardware

1) *Robot computers:* To meet all the requirements in robot control and features described in section II, high CPU power is required. In order to achieve the required computational power, two computers are used on the robot. The computers are interconnected through an ethernet network for informa-

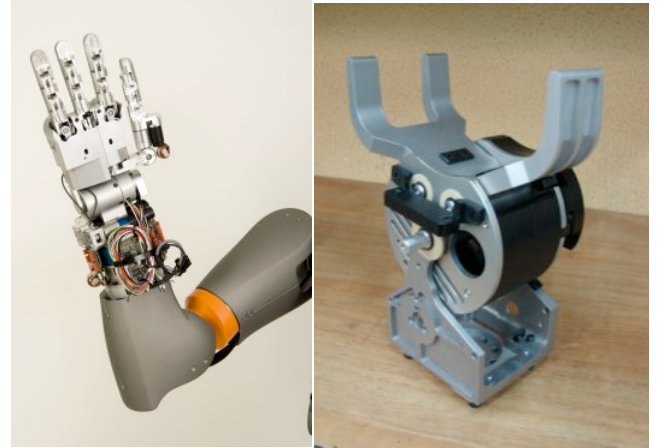


Fig. 5. Some pictures of the hands. (left) Fingered hand. (right) Gripper hand.

tion share. The use of two computers allowed the division of the tasks in two types, control and multimedia, allowing the adaptation of the computer requirements to each type of task.

The first computer is called the *control computer* and is based on a Geode 500Mhz. This computer is designated to perform control tasks on real time, which include walking behavior, manipulation of objects by controlling the arms and the fingers, and any motion related activity, like climbing stairs or sitting down on chairs.

The second computer is the *multimedia computer*. It is



Fig. 4. Reem-B sitting down and stand up from a chair sequence

based on a Core 2 Duo at 1.66 Ghz. This computer is used for navigation in real time, artificial vision routines (object identification and location, face recognition), voice recognition and synthesis, and other additional tasks like maintenance of the agenda, telepresence activities or robot behavior coordination.

The electronic architecture of the computers is that of a PC-104+. This stack provides expandability to the CPU-board while keeping reduced dimensions. The stack of the control computer is composed of 2 PC-104 expansion boards: a double CAN controller, which can achieve a bandwidth of about 2 Mbps, and two 6-axis force/torque sensor boards, two on the feet and two on the hands. The stack of the multimedia computer has an IEEE-1394 communication board, used to receive images from the cameras, several USB ports for wireless connectivity and extra connectivity. Lasers and ultrasonic sensors are also connected to this computer.

The whole system runs on Linux for embedded systems, including a patch for real time control. This architecture allows the programmers to develop the robot software in a desktop computer and, once the application is stable, cross-compile and upload it to the robot computer.

2) *Motor Control Board*: Reem-B is equipped with 39 motor control boards of different size, almost one for the control of each DoF (except in the hand where one board controls two DoFs). Motors below 150 watts are powered by custom boards of very small dimensions (about 60 mm including connectors). In order to reduce the weight and complexity of cabling, boards are placed as close as possible to the motor to drive. Each board includes its own DSP, which allows position control with current limitation, and micro-interpolation of the target position given by the main computer, all of this at a frequency of 100 Hz.

Current limitation integrated in the position control loop is an important feature, having two main advantages: it protects the motor, the reducer and the driver itself in case of unexpected forces (i.e. when a joint is stacked in a fixed position), and it makes possible to simulate torque control on the joint, virtually reducing the impedance on the joint.

3) *Sensors*: The sensors are basically used to detect the current state of the robot body. For instance, the feet of the robot are equipped with 6 axis force/torque sensors to measure the real position of the center of pressure; this

information can be integrated in the control loop of the robot to guarantee more equilibrium during dynamic walking.

Body tilt measurement is also necessary for stable walking, as it contributes to keep torso upright. A new tilt sensor has been developed for this robot. The information from one accelerometer/gyroscope is processed by a DSP to get reliable and robust measurements of tilt angles.

One typical application is a *follow the human* behavior, where the human can guide the movement of the robot by acting on its arm. Another application case is to generate a reactive *shake hands* behaviors, where the robot moves the arm in reaction to a hand shake. Hands in the arms are equipped with IR sensors used to detect objects between fingers.

Located on the feet there are two laser sensors used for autonomous navigation. They allow the creation of a map, localization on it, and even path planning with obstacle avoidance. An extra laser is mounted on the robot mouth. It allows the detection of close obstacles and especial furniture obstacles like tables or chairs with strange shapes. Additionally, the robot is equipped with six ultrasonic sensors mounted on its torso that are mainly used to detect people or obstacles around the robot.

Finally, in order to interact efficiently with people, the robot has a stereo camera and a stereo microphone, both located on its head.

4) *Batteries*: The robot is powered by a Lithium-polymer battery. This technology was preferred because of the higher power density and its superior weight over dimension rate when compared to other technologies like Nickel-Metal.

The battery pack has a rated voltage of approximately 45 volts. One single pack has power enough to provide energy to all robot elements. However, converters are required to adapt the voltage to different levels. A first group of converters powers the larger motors of the robot, located on the legs. A second and third pack of 28 volts are used for the motors of the upper body and the computers. Summarizing, the robot's electronics, including motor control boards, sensors and the PC-104+ stack, consumes about 100 watts per hour.

The total energy that can be provided by the batteries is about 670 Watts/hour, and experiments reveal that autonomy approaches the time of 120 minutes walking continuously. This is partly due to the power amplifiers of the motors,



	Reem-B	HRP-2	Asimo	Wabian
Watt/hour	670	710	384	365
W/h normalized by weight	16	12.2	7.4	5.6

TABLE II

AUTONOMY COMPARISON, BASED ON THE CAPACITY TO PROVIDE ENERGY

which work effectively in four-quadrants and make possible power regeneration: during braking, in fact, current flows other way around, recharging the batteries.

In table II we compare the batteries of our robot with those of Asimo, HRP-2 and Wabian; one of the characteristic compared among the robots is the energy provided normalized by the weight of the robot. Even if autonomy is not linearly proportional to this quantity, it can be expected that a higher ratio would correspond to longer operational time.

### C. Integrated software

1) *Operating system*: The requirements for the operating system in the project are quite restrictive. It must combine the traditional embedded features such as small footprint and robustness with real time capabilities. It must cope with the eventuality of power being cut off at any time without prior notice and the boot strategies for the platform must be reliable and without mechanical parts.

The operating system chosen was Linux. An embedded distribution was used in order to remove unnecessary drivers from the kernel and to reduce the applications size (stripping the libraries and eliminating debug information). A key feature of the Linux kernel was the availability of the source code. Kernel preemption and low latency patches were applied in order to provide a better real time response.

The initialization of the kernel was modified to generate an image bootable directly from a USB pen drive or from a LAN using the Etherboot mechanism. This image loads completely in memory the whole operating system and the applications, which prevents any corruption of the file system in the event of a power being cut off.

2) *Behavior architecture*: The behavior of the robot is coordinated by a hierarchical and asynchronous finite state machine (FSM). Our software framework embeds into the FSM robotics specific capabilities. The FSM states (i.e. the task steps) can be sequential or concurrent. They state transitions can also be triggered by external events, like receiving a TCP-IP message, a vocal command or by the detection of a person's face. Behaviors defined in such way can be built to form complex tasks like looking for a bottle and serve a drink.

3) *Vision software*: The robot vision software is used to detect color regions, and, using the stereo information, obtain the distance of different objects in the field of view of the camera. The algorithm speed has been subsequently increased by using libraries accelerated by hardware using special SSE instructions of the Intel processor.

Color segmentation is based on a double threshold: first a nearly pure color is segmented, then contiguous regions

with larger and more permissive threshold are merged to the segmented object. In this way, the system has a more robust detection in structured environment where the objects to recognize have just one color, like in balls, bottles or some landmarks.

The vision software is also used to detect, and recognize faces. This ability is used to generate a *follow the face behavior*, or to maintain an agenda associated to each person.

In some special cases, the images captured by the camera are used as *visual tags*. Visual tags are stored in memory during the map creation phase and associated to a specific place in the map. When the robot starts localizing and moves around the map, visual tags are identified helping to reduce localization errors.

4) *Voice software*: The voice software is composed of two different parts: the speech recognizer and the speech synthesizer. Both systems are integrated in the robot from existing commercial solutions. They are able to interact with the FSM, generating recognition events, or being triggered by the FSM to synthesize sentences.

5) *Navigation software*: Localization and mapping abilities are based on the use of particle filters that are fed by laser and odometry information. For the implementation of the SLAM feature, an algorithm called DP-SLAM was selected [20]. This algorithm allows an almost real-time generation of maps, without requiring posterior post-processing. This means that our robot creates its environment map at the same time that it walks around it, and no further processing is required. Once the map is completed, the robot directly changes its operation mode to localization, and starts using the map for localization and path planning, without requiring any additional step.

The localization algorithm is based on a typical Monte-carlo particle filter [21], [22], with kidnapping capabilities. Hence, the robot can perform global localization and start localizing itself on the map just straight after map completion without any human intervention. Once the robot is able to localize itself on the map, the robot can be asked to go to any place in the map, autonomously, while avoiding obstacles on its path and replanning when necessary.

As it is shown in section IV-B, the current state of the navigation system can be monitored on a GUI running on an external computer wirelessly connected to the robot. Nevertheless, it is not mandatory to use that GUI to allow the robot navigate autonomously since all the robot programs are executed onboard the robot, and any required interaction is made by voice.

## IV. ROBOT PERFORMANCE

This section shows some robot performance results in common daily life activities<sup>1</sup>.

### A. Robot movement

Different control algorithms allow the robot to perform different motions. For instance the walking algorithm allows

<sup>1</sup>Videos available at [www.pal-robotics.com](http://www.pal-robotics.com)

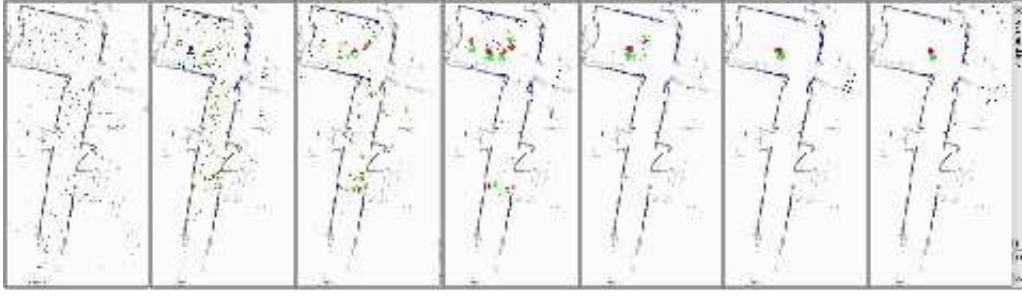


Fig. 6. Reem-B localization sequence on a previously created map



Fig. 7. Reem-B manipulation sequence to serve a drink

the robot to walk at peak speeds of 1.5 Km/h in forward direction. Backwards motion is also supported at slower speeds. Complex motions like turning on the spot, curve trajectories (i.e. , turning while walking, forward or backwards) and lateral steps are also possible.

Other examples of motion performed by the robot include a climbing stairs behavior, with a current limitation in the maximum step size to climb of 10 cm. The robot is also able to sit down on a chair and stand up from it.

### B. Autonomous navigation

Using two lasers one on each foot of the robot, Reem-B is able to construct a map, and then, localize on it, perform planned trajectories and avoid obstacles on its trajectory. The robot is also able to replan trajectories on the fly, when unexpected obstacles appear on its path. The whole process is performed onboard the robot, but it is possible to observe what is the current map that the robot is using, the trajectory planned, etc, connecting to the robot from an external computer. Figure 6 shows some captures of the maps created by Reem, while performing localization, or generating some trajectories.

Even if navigation is mainly guided by laser data, it can be helped by vision. The robot uses the stereo camera to store images during map construction of special places of the map (detected as visual landmarks). The location of those landmarks is used later when the degree of location confidence decreases below a certain level. At that point, the robot tries to recognize any of the landmarks and reduce its error in localization whenever one is found.

### C. Visually guided manipulation

The combination of hands manipulation, arms control and vision allows for the control of visually guided behaviors. We have applied this to the behavior of serving a drink, where the grasping and serving movements are guided only by stereo vision. The gripper hand is used to grasp the glass and move it to a convenient place. Then the same hand is used to grasp the bottle and serve the drink on the glass.

Another application is playing chess. In this case, the fingered hand is used to grasp the pieces and move them to the desired place on the chessboard. Then the robot stops its clock indicating a turn change. The whole process is visually guided.

## V. DISCUSSION

Humanoid robots have been highly criticized, arguing that there is no need to have such a complex robot to perform tasks at home or to help people. Some researchers advocate for more specialized types of robots, specialists on performing one type of task with a lower degree of complexity. However, in most cases, the decrease in complexity goes along the necessity to have a very concrete environment to work correctly. Therefore, in order to help in all different kinds of tasks and home environments, it would be required to have a legion of different robots to perform all those tasks.

Instead, our team proposes the use of a more general robot. The idea is to solve the robotic challenge by creating complex robots that can handle current home environment setups, instead of creating suitable setups for simpler robots.

Hence we pursue the creation of humanoid robots to help people.

A humanoid allows a quick adaptation of the robot to any type of home or office environment, and can, off-the-shelf, act upon the human stuff already in place. If correctly designed, no extra adaptation is required. A humanoid robot can move in the same way as a human on that environment. Furthermore, it can pick loads up by itself and transfer to other zones of the environment, while avoiding typical obstacles like stairs or small ground differences between zones.

The challenge is then on designing a good humanoid robot. However, we are still far from that. Too many challenges need to be overcome in all the areas involved in the creation of a humanoid, including mechanics, motors, manipulation or vision and speech recognition. Among all those problems to solve, one of the more challenging is that of the autonomy of the robot, that is, the ability to behave by itself and create its own agenda. In this sense, the Reem-B has a superior level related to previous humanoid robots of its size. Reem-B is able to safely move by itself on an indoor environment, during a long period of time, and without needing external components or computer power. The whole robot is self-contained.

Nevertheless, the robot is still far from complete. Challenges for the future include a socially based human-robot interaction mechanism, improved object recognition under different light conditions, and a behavior based core for a life long autonomous control of the robot. Also, some safety measures need to be incorporated, including a safe fall down mechanism, recovering from falls and emergency shutdowns.

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